# COURSE DESCRIPTION CARD - SYLLABUS

Course name
Linear algebra [S1Cybez1>ALIN]

## Course

Field of study
Cybersecurity

Year/Semester
1/1

Area of study (specialization)
–

Profile of study
general academic

Level of study
first-cycle

Course offered in
Polish

Form of study
full-time

Requirements
compulsory

## Number of hours

Lecture
30

Laboratory classes
0

Other
0

Tutorials
30

Projects/seminars
0

## Number of credit points

5,00

## Coordinators

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## Lecturers

## Prerequisites

1.Mathematical knowledge from the secondary school 2. Ability to solve problems and mathematical modeling at the level of secondary school.

## Course objective

1 To provide students with basic knowledge of mathematics in complex numbers, polynomials, algebraic structures, matrix calculus, linear spaces and linear operators, and analytic geometry in space. 2. To develop in students the ability to solve simple mathematical modeling problems by applying the methods of abstract algebra and linear algebra. 3. To develop in students the ability to describe relationships and relations using the concepts of algebra

## Course-related learning outcomes

Knowledge:
1. has an extended and in-depth knowledge of mathematics useful for formulating and solving complex computer tasks concerning, among other things, programming in logic, formal specification and verification of software, as well as tasks in physics, electrical engineering fundamentals, electronics fundamentals and automation fundamentals [K1_W01],

2. has knowledge of matrix calculus, operations on matrices, determinants of matrices, methods of calculating inverse matrices, application of matrix calculus to solve systems of linear equations [K1_W01],
3. has knowledge of basic algebraic structures - monoids, groups, rings and fields [K1_W01],
4. has knowledge of n-dimensional linear space, linear subspace, base of space, change of base [K1_W01],
5. has detailed knowledge of selected branches of mathematics (needed to understand selected branches of physics, fundamentals of electrical engineering, and fundamentals of electronics and telecommunications) [K1_W01],
6. has knowledge of complex numbers, operations on complex numbers, forms of complex numbers and their applications [K1_W01],
7. has knowledge of operations on vectors in three-dimensional space [K1_W01],
8. has knowledge of operations on polynomials and roots of polynomials, also in the complex domain [K1_W01].

Skills:
1. is able to plan and carry out computer simulations, interpret the results obtained and draw conclusions, and in particular, as part of these activities, is able to perform operations on complex numbers, find real and complex roots of certain types of polynomials [K1_U05],
2. is able to use analytical methods to formulate and solve computer tasks [K1_U05],
3. can perform operations on matrices, determine the inverse matrix by the method of elementary operations, calculate the determinant of a matrix, solve a system of linear equations by the Gauss method [K1_U05],
4. recognize algebraic structure, can apply monoid and group structure to describe the states of devices or processes [K1_U05],
5. know how to determine the dimension of linear space and subspace, know how to perform the change of the basis of linear space, know how to solve the matrix eigenproblem [K1_U05],
6. can perform operations on vectors in three-dimensional space and apply the methods of vector calculus [K1_U05].

Social competences:
1. is able to appropriately determine the priorities for the realization of a task defined by himself or others, including the ability to think and act in a rigorous manner in the area of process description in technical and scientific sciences [K1_K01 ].

## Methods for verifying learning outcomes and assessment criteria

Learning outcomes presented above are verified as follows:

Lecture
-assessment of knowledge and skills on a written exam testing knowledge of notions, ability to carry out simple proofs and illustrate theory with examples (short practical tasks are also possible).
-gaining additional points for activity during classes, including presentation of talks (discussing additional aspects of the issues, in particular the application of the discussed theory in other sciences or referring to the location in the history of mathematics) and for comments on the improvement of teaching materials.
Passing threshold: at least 50% of the points. Issues for the exam, on the basis of which questions are developed, will be sent to students by e-mail using the university's electronic systems.
Tutorials:
-continuous assessment - bonus activity (additional points) manifested in discussion and in cooperation in solving practical tasks,
--continuous assessment - rewarding the increment of skills in using the learned techniques,
--gaining additional points for activity during classes, including presentation of talks (discussing additional aspects of the issues, in particular the application of the discussed theory in other sciences or reference to the location in the history of mathematics) and for comments on the improvement of teaching materials
-active participation in consultations that deepen knowledge and guide further work.
The knowledge acquired in the exercises is verified by two tests implemented in about the 7th and 15th week (alternatively, a short test at each tutorial). Passing threshold: at least 50% of the points.
The course completion rules and the exact passing thresholds will be communicated to students at the beginning of the semester through the university’s electronic systems and during the first class meeting

(in each form of classes).

## Programme content

1. complex numbers
2. matrix calculus and its applications
3. basic algebraic structures
4. linear spaces and linear operators, eigenproblem of linear operator (matrix).
5. operations on vectors

## Course topics

Lecture
Complex numbers
1 The algebraic, trigonometric and exponential forms of a complex number.
2. Operations on complex numbers: addition, multiplication and division in algebraic and trigonometric form.
3. Conjugate number and properties.
4. Properties of modulus of product, quotient of complex numbers.
5. Properties of the argument of the product, quotient and power of complex numbers .
6. Moivre's formula.
7. Evolution of complex numbers in trigonometric form - theorem and geometric interpretation.
8. Algebraic equations in the complex domain and Gauss' theorem.
Matrix calculus part 1.
9. definition of matrix, types of matrices, examples.
10. Operations on matrices: addition of a matrix, multiplication of a matrix by a number, multiplication of a matrix by a matrix (definitions and properties).
11. Definition of a system of linear equations. Gauss elimination method.
12. Laplace's expansion theorem for the determinant. Properties of determinants.
13. Cramer's system-definition and theorem.
14. Inverse matrix- definition and properties. Two theorems on the determination of the inverse matrix.
15. Interpretation of the system of equations (solution) in terms of matrix (inverse matrix).
Basic algebraic structures
16. Monoid, group (abelian). Cyclic groups.
17.Homomorphism, types of homomorphisms of groups.
18.Rings, ring ideal, homomorphism of rings. Fields - examples.
Linear spaces
19. Definition of linear space, examples.
20. linear combination.
21. linear subspace.
22. definition of linear span, examples.
23. linearly dependent vectors and linearly independent vectors.
24. dimension of linear space.
25. basis of linear space.
26. Theorem-equivalent condition for a system B={e $_1$ ,e $_2$ ,...e_{n}} to be a basis.
Matrix calculus part 2
27. row space of matrices- definition and theorem of row-equivalent matrices.
28. matrix rank- definition and equivalent formulations.
29. Kronecker-Capelli theorem and the conclusion on the number of solutions.
30. homogeneous systems- definition and discussion of the number of solutions based on Kronecker-Capelli's Theorem.
Linear transformations, values and eigenvectors of a linear operator.
31. definition of linear operator, examples. Kernel and image of a linear operator.
32. Theorem of representation of linear operator by matrix.
33. Definition of eigenvalue and eigenvector. Eigenspace subspace and spectrum.
34. characteristic equation and characteristic polynomial of a matrix.
35. the Cayley-Hamilton theorem.
Analytic geometry
36 Vector. Vector length, directional cosines.
37. Rectangular Cartesian coordinate system in space.

38. parallel vectors, perpendicular vectors.
39. addition of vectors. Multiplication of a vector by a number.
40. scalar product, vector and mixed product of vectors.
41. parametric equation of a straight line in space.
42. equations of a plane in space.
Exercises
Solving practical tasks using the material presented in the lecture

## Teaching methods

I. Lectures
1. lecture conducted on the blackboard in an interactive manner with the formulation of questions to a group of students, the lecture supplemented by a computer presentation.
2. the activity of students (preparation of historical papers on mathematicians related to the presented material, papers on applications of algebra in engineering sciences, presentation of proofs left to do on their own) during the class is taken into account when assigning the final grade,
3. during the lecture to initiate discussion,
4. theory presented in connection with the current knowledge of students from previous lectures.
II. Tutorials
1. solving sample tasks on the blackboard
2. detailed review of solutions to tasks by the instructor of exercises and discussion of comments.

## Bibliography

Basic:
1. W. Leksiński, I. Nabiałek, W. Żakowski, Matematyka. Definicje, twierdzenia, przykłady, zadania, seria EIT, WNT Warszawa 1992 (i późniejsze)
2. W.J. Gilbert, W.K. Nicholson, Algebra współczesna z zastosowaniami, WNT Warszawa 2008
3. M. Grzesiak, Liczby zespolone i algebra liniowa, Wydawnictwo PP Poznań 1999
4. S. Przybyło, A. Szlachtowski, Algebra i wielowymiarowa geometria analityczna w zadaniach, WNT Warszawa 1992 (i późniejsze)
5. T. Jurlewicz, Z. Skoczylas, Algebra liniowa 1, Wrocław 2003.
6. T. Jurlewicz, Z. Skoczylas, Algebra liniowa 2, Wrocław 2005.

Additional:
1. H. Arodź, K. Rościszewski, Zbiór zadań z algebry i geometrii analitycznej dla fizyków, PWN, 1990.

## Breakdown of average student's workload

| | Hours | ECTS |
|---|---|---|
| Total workload | 130 | 5,00 |
| Classes requiring direct contact with the teacher | 60 | 2,50 |
| Student's own work (literature studies, preparation for laboratory classes/ tutorials, preparation for tests/exam, project preparation) | 70 | 2,50 |